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TECHNICAL MEMORANDUM

ATTITUDE CONTROL OF DYNAMICALLY UNBALANCED ARTIFICIAL GRAVITY SPACE STATIONS

Bellcomm

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ABSTRACT

Effects of dynamic unbalances are determined for an axially symmetrical, dual-spin space station consisting of a rotating artificial gravity section and a controlled, despun zero gravity section. Configurations with either a rigid interconnection or a low coupling flexible interconnection between sections are considered. Amplitudes of motion and control system requirements are compared for these configurations. Both configurations are found to experience coning motions. For large space stations with rigid interconnection, CMGs are found impractical for reducing coning motions to within allowable limits for many experiments and special systems such as active mass balancing are necessary. However, for a flexible interconnection, coning motions of the despun section are substantially reduced and CMGs could be utilized to reduce motions to acceptable levels.

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FROM: R. A. Wenglarz

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TECHNICAL MEMORANDUM

1.0 Introduction

For the future, NASA is considering a large, manned space base which will provide artificial gravity and zero gravity environments simultaneously in separate sections.[1] Artificial gravity will likely be obtained by spinning of the artificial-g section so that objects within experience a radial acceleration toward the spin axis. However, if the spin axis of this section is not coincident with a principal axis of inertia, large inertia torques are imparted to the space station. These torques can be much greater in magnitude than gravity gradient, aerodynamic, and other environmental torques and have a corresponding greater effect on attitude motions.

Because of uncertainties in construction and because future space stations will provide freedom of movement for crew members and equipment, the location of principal axes of inertia for the rotating section will not only be imprecisely known but also will be continually changing. Consequently, deviations of the spin axis from a principal axis of inertia and resulting disturbance torques are inevitable.

Following is a discussion of the effects of small spin axis misalignments on the attitude motion and control moment gyro (CMG) requirements for two axially-symmetric, dual-spin space base configurations. One configuration involves a rigid interconnection between the despun and spinning sections. The other involves an interconnection which transmits only low levels of torque normal to the symmetry axis. The more complex low coupling interconnection is considered because it could, as suggested in Reference 2, substantially reduce attitude motions of the despun section.

A comparison is given here of the amplitudes of motion and also CMG requirements for the rigid and low coupling interconnections. Details of the system analysis are covered in the analysis section and the Appendices.

2.0 Results

A dynamical representation for a space base configuration is shown in Figure 1. Low coupling interconnection between the spinning and despun sections is characterized by the gimbal system shown in Figure 2. The gimbal system represents interconnections for which relative motions of the satellite sections maybe adequately described by rotations alone. Gimbal axes torques are assumed to have components proportional to gimbal angle deflections and gimbal rates. Rigid interconnection between bodies is characterized by taking the constant of proportionality associated with gimbal deflections to infinity.

System details and solutions to the space base equations of motion are given in the analysis section. Equations of motion for the spacecraft are formulated in Appendix A. A method of successive approximations for integrating the equations of motion and a means for establishing asymptotic stability of motions are given in Appendix B. CMG requirements necessary to provide the prescribed control torques on the despun section are found in Appendix C.

For either a rigid interconnection or low coupling interconnection, the symmetry axis of the despun section traces a right circular cone in space. CMGs cannot significantly decrease the coning amplitude unless composite CMG spin angular momentum is of the order of a limiting value. Both the coning angle and CMG momentum capacity necessary to reduce this angle are substantially less for the low coupling interconnection than for the rigid interconnection.

For example, a space base configuration* with rigid interconnection which has been suggested for study by NASA[1(a)] experiences coning with a half angle of 0.54° for 0.5° spin axis misalignment and low levels of attitude control.** This value is unacceptable for fine pointing space station experiments[3] rigidly mounted. Also, resulting accelerations are unacceptable for crystal growth and other experiments. To significantly reduce the amplitude of motion would require the momentum capacity of more than 730 CMGs of the Skylab type (2300 ft-lb-sec per CMG).

For the same space base configuration with a low coupling interconnection, the despun section experiences coning with a 0.037° half angle for low levels of attitude control. To significantly reduce this amplitude would require the momentum capacity of only four CMGs of the Skylab type. These results are for a gimbal design in which the gimbal axes intersect one foot or less from the mass center of the spinning section and for gimbal stiffness and damping coefficients adjusted to suitably low levels. It appears probable that in practice coning

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*This configuration consists of a Y-shaped rotating section attached to a hub with a despun section.

**This corresponds to CMG spin angular momentum so small that the CMGs do not significantly influence the amplitude of coning motion.

amplitudes and CMG requirements could be reduced further, perhaps an order of magnitude, for gimbal torque coefficients adjusted to lower levels. However, verification of stability of the resulting motion requires more general analytic techniques. The most efficient of these involves computer applications which require specific assumptions on system parameters. Since the main purpose here is to demonstrate possible advantages of low coupling interconnection over rigid interconnection, such detailed computer work is beyond the scope of this study.

For space base with rigid interconnection, CMG's do not appear practical in reducing coning motions induced by mass unbalances. Since reaction jet attitude control alone is also impractical for long duration orbital missions, special systems such as active mass balance systems[2] would have to be employed. Such systems are complex and require a substantial weight penalty. A low coupling interconnection however may provide a passive system for alleviating coning effects that is relatively simple and lower in weight. Experiments requiring small motions could be housed in the despun section which could be controlled by a reasonable number of CMG's while the spinning section experiences its coning motion. Since restrictions on motions (wobble) resulting from crew comfort requirements are substantially less than those resulting from experiment requirements, the coning motion of the spinning section might well be acceptable. For the space base example given above, motions of the spinning section are within limits of current estimates[4] for crew tolerance.

In conclusion, the results of the analysis section may be applied for investigating other aspects of attitude control of dual-spin satellites than are considered here. For example, maximum acceptable misalignments for specified CMG momentum and torque capacity and effects of bearing assembly flexibility can be determined.

ANALYSIS

System Description

The space station dynamical model to be considered is shown in Fig.1 and consists of two sections attached by a massless gimbal arrangement shown in Fig.2. The satellite is stabilized by CMG's providing three axis control on one of the sections. The other section is assumed driven at a constant rate about a line which is not a principal axis of inertia for that body. (The results for amplitudes of satellite motion and CMG requirements with B driven are adequate piecewise representations for B freely spinning, as discussed in the following section.)

System details and terminology are given as follows: The spinning section is termed body B and the other section is termed body A. Both are axially symmetrical with principal moments of inertia for their mass centers denoted by B_3 and A_3, respectively, in the directions of the symmetry axes and B_1 and A_1, respectively, in directions normal to the symmetry axes. For body B, the mass is designated M_B, the mass center is designated B*, and a righthanded, mutually orthogonal set of unit vectors parallel to principal axes of B at B* is designated by $\underline{b}_1, \underline{b}_2, \underline{b}_3$. Similar quantities for body A are mass M_A, mass center A*, and unit vectors $\underline{a}_1, \underline{a}_2, \underline{a}_3$, parallel to principal axes at A*.

An inertially fixed set of unit vectors $\underline{a}_{10}, \underline{a}_{20}, \underline{a}_{30}$, is defined by the initial orientation of $\underline{a}_1, \underline{a}_2, \underline{a}_3$ at time t=0. The orientation of $\underline{a}_1, \underline{a}_2, \underline{a}_3$ (and consequently A) at some subsequent time t is described with respect to $\underline{a}_{10}, \underline{a}_{20}, \underline{a}_{30}$, by a 1,2,3 sequence of three axis Euler angles ϕ_1, ϕ_2, ϕ_3. Also, the CMG control torque on A is assumed related to ϕ_1, ϕ_2, ϕ_3, by

$$\underline{T}^{C\,A} = -[(K_0\phi_1+K_1\dot{\phi}_1)\underline{a}_1 + (K_0\phi_2+K_1\dot{\phi}_2)\underline{a}_2 + (K_{03}\phi_3+K_{13}\dot{\phi}_3)\underline{a}_3] \qquad (1)$$

Body B is driven at a constant rate ω about a drive axis which is slightly misaligned from the principal axis in the direction of $\underline{b}_3$. Let $\underline{b}_1'$, $\underline{b}_2'$, $\underline{b}_3'$ be a right handed set of mutually orthogonal unit vectors fixed in B with $\underline{b}_3'$ in the direction of the drive axis and let the transfer relations between $\underline{b}_i$ and $\underline{b}_j'$ be given by

$$\underline{b}_i = \sum_{j=1}^{3} b_{ji}\, \underline{b}_j' \qquad (2)$$

The drive axis of B intersects B* and point 0 which is the common intersection of the symmetry axis of A and the axes of the gimbal connecting A and B. A* and B* are located at distances ℓ_A and ℓ_B, respectively, from 0. Rotations θ_1 about gimbal axis G_1 and θ_2 about gimbal axis G_2 describe the orientation of the drive axis of B with respect to $\underline{a}_1$, $\underline{a}_2$, $\underline{a}_3$ as shown in Fig. 2. $\underline{c}_1$, $\underline{c}_2$, $\underline{c}_3$ designate unit vectors such that $\underline{c}_3$ is parallel to the drive axis, $\underline{c}_2$ is parallel to G_2, and $\underline{c}_1$ completes the mutually orthogonal right-handed set. Motion of A relative to B is resisted by gimbal axes torque ${}^{G}\underline{T}^{A}$ on A with components ${}^{G}T_i^{A}$ along G_i given by

$${}^{G}T_i^{A} = C_0\theta_i + C_1\dot{\theta}_i \qquad i = 1,2 \qquad (3)$$

These torques represent gimbal stiffness and dissipation.

SYSTEM ANALYSIS

Equations of Motion

Gravity gradient and other environmental torques may be neglected in the formulation of equations of motion because, for rotation rate ω much greater than orbital rate, their magnitudes are much smaller than the magnitude of torque associated with misalignment of the spin axis of B.

Although body B is assumed driven at a constant rate, for some applications it could be practical to assume zero torque on B about the spin axis so that B is freely spinning. This implies L defined in Eq. (A-13) vanishes. Eq. (A-7) shows that the time derivative of the component along $\underline{c}_3$ of the inertial angular velocity of B is very small for small spin axis misalignments. Then the spin speed of B is changing very slowly and could be considered constant over some interval of time. Although significant changes in mean spin speed can occur over very long time intervals, the following results for amplitudes of satellite motion and CMG requirements with B driven at a constant speed likely yield adequate piecewise representations with B freely spinning. Spin speeds of B could be considered constant over successive long intervals of time with variation in constant values between intervals computed by Eq.(A-7).

The three Euler angles ϕ_1, ϕ_2, ϕ_3, which describe the orientation of body A relative to inertially fixed axes, the two gimbal angles θ_1 and θ_2, and the prescribed rotation ωt of B about its spin axis provide a complete description of the satellite orientation. Linearization about initial state $\phi_i = \theta_j = 0$, $i = 1,2,3$; $j = 1,2$ of the equations of motion formulated from Eq. (A-6) - (A-13) of Appendix A yields

$$\ddot{\phi}_1[B_1+Bb_1+M\ell_B(\ell_A+\ell_B) + B(-b_2 s2\omega t+b_3 c2\omega t)] + \ddot{\phi}_2 B(b_3 s2\omega t+b_2 c2\omega t)$$

$$+ 2\dot{\phi}_3 Bb_{33}(b_{13}c\omega t-b_{23}s\omega t)$$

$$+ \ddot{\theta}_1[B_1+Bb_1+M\ell_B^2+B(-b_2s2\omega t+b_3c2\omega t)] + \ddot{\theta}_2B(b_3s2\omega t+b_2c2\omega t)$$

$$- \dot{\phi}_1\omega B(b_3s2\omega t+b_2c2\omega t)$$

$$+ \dot{\phi}_2\omega[B_3-Bb_1+B(-b_2s2\omega t+b_3c2\omega t)] - 4\ddot{\phi}_3\omega Bb_{33}(b_{23}c\omega t+b_{13}s\omega t)$$

$$+ \dot{\theta}_1[C_1-\omega B(b_3s2\omega t+b_2c2\omega t)]$$

$$+ \dot{\theta}_2\omega[B_3-Bb_1+B(-b_2s2\omega t+b_3c2\omega t)] + \theta_1C_0 = 2\omega^2Bb_{33}(b_{23}c\omega t+b_{13}s\omega t) \tag{4}$$

$$\ddot{\phi}_1B(b_3s2\omega t+b_2c2\omega t) + \ddot{\phi}_2[B_1+Bb_1+M\ell_B(\ell_A+\ell_B) + B(b_2s2\omega t-b_3c2\omega t)]$$

$$+ 2\ddot{\phi}_3Bb_{33}(b_{23}c\omega t+b_{13}s\omega t)$$

$$+ \ddot{\theta}_1B(b_3s2\omega t+b_2c2\omega t) + \ddot{\theta}_2[B_1+Bb_1+M\ell_B^2+B(b_2s2\omega t-b_3c2\omega t)]$$

$$- \dot{\phi}_1\omega[B_3-Bb_1+B(b_2s2\omega t-b_3c2\omega t)]$$

$$+ \dot{\phi}_2\omega B(b_3s2\omega t+b_2c2\omega t) + 4\dot{\phi}_3\omega Bb_{33}(b_{13}c\omega t-b_{23}s\omega t)$$

$$- \dot{\theta}_1\omega[B_3-Bb_1+B(b_2s2\omega t-b_3c2\omega t)]$$

$$+ \dot{\theta}_2[C_1+\omega B(b_3s2\omega t+b_2c2\omega t)] + \theta_2C_0 = -2\omega^2Bb_{33}(b_{13}c\omega t-b_{23}s\omega t) \tag{5}$$

$$2\ddot{\phi}_1Bb_{33}(b_{13}c\omega t-b_{23}s\omega t) + 2\ddot{\phi}_2Bb_{33}(b_{23}c\omega t+b_{13}s\omega t) + \ddot{\phi}_3(A_3+B_3-2Bb_1)$$

$$2\ddot{\theta}_1Bb_{33}(b_{13}c\omega t-b_{23}s\omega t) + 2\ddot{\theta}_2Bb_{33}(b_{23}c\omega t+b_{13}s\omega t) + \dot{\phi}_3K_{13}+\phi_3K_{03} = 0 \tag{6}$$

$$\ddot{\phi}_1[A_1+M\ell_A(\ell_A+\ell_B)] + \ddot{\theta}_1 M\ell_A\ell_B + \dot{\phi}_1 K_1 + \phi_1 K_0 - \dot{\theta}_1 C_1 - \theta_1 C_0 = 0 \tag{7}$$

$$\ddot{\phi}_2[A_1+M\ell_A(\ell_A+\ell_B)] + \ddot{\theta}_2 M\ell_A\ell_B + \dot{\phi}_2 K_1 + \phi_2 K_0 - \dot{\theta}_2 C_1 - \theta_2 C_0 = 0 \tag{8}$$

where

$$B = \frac{B_3-B_1}{2} \qquad M = \frac{M_A M_B}{M_A+M_B} \qquad b_1 = b_{13}^2+b_{23}^2 \qquad b_2 = 2b_{13}b_{23}$$

$$b_3 = b_{13}^2-b_{23}^2 \tag{9}$$

and $s2\omega t = \sin 2\omega t$, $c2\omega t = \cos 2\omega t$.

Although linearized, these equations involve periodic coefficients and standard techniques for a closed form solution are not available. However, they can be written in a standard matrix form that will be seen to have a stable steady state periodic solution which may be represented through a method of successive approximations for small spin axis misalignments.

Now, establishment of a new independent variable,

$$\tau = \omega t \qquad \frac{d}{dt}(\ ) = \omega\frac{d}{d\tau}(\ ) = \omega(\ )' \tag{10}$$

the writing of Eqs.(4)-(8) in terms of τ, solution of these equations for ϕ_1'', ϕ_2'', ϕ_3'', θ_1'', θ_2'' and definition of

$$x_i = x_{i+5}', \qquad i=1,\dots,5 \qquad x_{i+5} = \phi_i, \qquad i=1,2,3$$

$$x_{j+8} = \theta_j, \qquad j=1,2 \tag{11}$$

along with

$$n_{11} = e_1k_1/\Delta \qquad n_{12} = e_2a/\Delta \qquad n_{13} = (ah_{12}+e_1h_{11})/\Delta$$

$$n_{14} = e_1k_0/\Delta \qquad n_{15} = (ah_{02}+e_1h_{01})/\Delta$$

$$n_{21} = k_1/\Delta \qquad n_{22} = e_2/\Delta \qquad n_{23} = (h_{12}+h_{11})/\Delta$$

$$n_{24} = k_0/\Delta \qquad n_{25} = (h_{02}+h_{01})/\Delta$$

$$n_{31} = k_{13} \qquad n_{32} = k_{03} \qquad \Delta = e_1-a \tag{12}$$

where

$$e_1 = (B_1+M\ell_B^2)/J_2 \qquad e_2 = B_3/J_2 \qquad h_{12} = C_1/\omega J_2 \qquad h_{02} = C_0/\omega^2 J_2$$

$$a = M\ell_A\ell_B/J_1 \qquad k_1 = K_1/\omega J_1 \qquad k_0 = K_0/\omega^2 J_1 \qquad h_{11} = C_1/\omega J_1$$

$$h_{01} = C_0/\omega^2 J_1$$

$$k_{13} = K_{13}/\omega J_3 \qquad k_{03} = K_{03}/\omega^2 J_3 \tag{13}$$

and

$$J_1 = A_1+M\ell_A(\ell_A+\ell_B) \qquad J_2 = B_1+M\ell_B(\ell_A+\ell_B) \qquad J_3 = A_3+B_3$$

$$d = B/J_2 \tag{14}$$

yields equations of motion in a standard matrix form,

$$x' = [D + F(\tau,\mu)]x + g(\tau,\mu) = f(\tau,x,\mu) \tag{15}$$

where μ is a two dimensional vector,

$$\mu = \begin{bmatrix} b_{13} \\ b_{23} \end{bmatrix} \tag{16}$$

Constant matrix D and column vector g are given by

$$
D = \begin{bmatrix}
-n_{11} & n_{12} & 0 & n_{13} & n_{12} & -n_{14} & 0 & 0 & n_{15} & 0 \\
-n_{12} & -n_{11} & 0 & -n_{12} & n_{13} & 0 & -n_{14} & 0 & 0 & n_{15} \\
0 & 0 & -n_{31} & 0 & 0 & 0 & 0 & -n_{32} & 0 & 0 \\
n_{21} & -n_{22} & 0 & -n_{23} & -n_{22} & n_{24} & 0 & 0 & -n_{25} & 0 \\
n_{22} & n_{21} & 0 & n_{22} & -n_{23} & 0 & n_{24} & 0 & 0 & -n_{25} \\
1 & & & & & & & & & \\
 & 1 & & & & & & & 0 & \\
 & & 1 & & & & & & & \\
 & 0 & & 1 & & & & & & \\
 & & & & 1 & & & & &
\end{bmatrix}
$$

$$
g = \begin{bmatrix} -ag_1 \\ -ag_2 \\ 0 \\ g_1 \\ g_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (17)
$$

where

$$g_1 = 2d_1(1-b_{13}^2-b_{23}^2)^{1/2}(b_{23}c\tau+b_{13}s\tau)$$

$$g_2 = 2d_1(1-b_{13}^2-b_{23}^2)^{1/2}(b_{23}s\tau-b_{13}c\tau) \tag{18}$$

$$d_1 = d/[\Delta + \text{terms of order 2 and higher in } b_{13}, b_{23}] \tag{19}$$

The matrix $F(\tau,\mu)$ is periodic in τ with period 2π with all elements proportional to first and higher powers in b_{13} and b_{23}. Consequently, for $b_{13}=b_{23}=0$ then $\mu=0$ and both $F(\tau,0)$ and $g(\tau,0)$ are zero.*

As discussed in Appendix B, Coddington and Levinson[5] have established a method of successive approximations for determining periodic solutions to systems of equations with a small real constant vector μ provided certain conditions are satisfied. These conditions applied to the problem at hand are
i) $x'=f(\tau,x,0)=Dx$ has no periodic solution of period 2π other than the trivial solution $x=0$, ii) $f(\tau,x,\mu)$ is analytic in (x,μ) for $|\mu|$ small. Furthermore, iii) if the real parts of the characteristic roots of matrix D are all negative, then the periodic solution to Eq. (15) is asymptotically stable for $|\mu|$ small. Then, in the steady state, for initial conditions $x|_{\tau=0}$ sufficiently small, all solutions to Eq. (15) approach the periodic solution represented by the method of successive approximations.

*This corresponds to the drive axis aligned along a principal axis of inertia.

Also, for the condition of statement iii) satisfied, statement i) is also satisfied and it is only necessary to test Eq.(15) with respect to statements ii) and iii).

Since f involves only the first power in x, it is obviously analytic, i.e., representable by a convergent power series, in x. It may be shown that f involves rational functions of b_{13}, b_{23} and $\sqrt{1-b_{13}^2-b_{23}^2}$, all of which are analytic for $|\mu|<<1$. Products, quotients and sums of these in f are also analytic since there results no poles (i.e., f bounded) for small $|\mu|$. Consequently, f is analytic and is representable by a convergent power series in (x,μ). Condition ii) is satisfied.

Routhian analysis[6] may be used to check condition iii) whether all characteristic roots of matrix D have negative real parts. However, for this case Routhian techniques require expansion of determinants of up to order ten. This process may be made manageable by taking advantage of the properties of the two satellite configurations of concern.

For rigid interconnection between satellite sections, C_0 is very large and n_{15} and n_{25} are very much greater than the other n_{ij} of matrix D.

As previously mentioned, a design that attenuates the effect of the motion of B on that of A is desired. Eqs.(7) and (8) show that the motion of A is decoupled from the motion of B if $M\ell_A\ell_B$, C_0, C_1 are zero. Although zero values cannot be achieved in practice, this suggests satellite design so that these are small. This low coupling design is characterized by regarding n_{12}, n_{13}, n_{15}, n_{23}, n_{25} as small compared to the other n_{ij} of matrix D. This corresponds to relatively small gimbal axis torques and to locating the intersection of the gimbal axes close to the mass center of A or B.

The above considerations enable one to neglect higher order powers of small parameters in the expansion of the Routhian determinants and conditions for all the characteristic roots of matrix D to have negative real parts are found to be

$$K_0,\ K_1,\ K_{03},\ K_{13} > 0 \tag{20}$$

for a configuration with rigid interconnection and

$$K_0,\ K_1,\ K_{03},\ K_{13},\ C_0,\ C_1 > 0 \tag{21}$$

for a configuration with low coupling interconnection.

Consequently, for Eq.(20) or (21) satisfied, the condition of statement iii) is satisfied and for $|\mu|$ and $x\big|_{\tau=0}$ small, the method of successive approximations may be applied to determine the steady state solutions of Eq.(15). Carrying terms to first powers in b_{13} and b_{23},

$$\phi_1 = q_{11}s\tau+q_{12}c\tau \qquad \phi_2 = q_{12}s\tau-q_{11}c\tau \qquad \phi_3=0 \tag{22}$$

$$\theta_1 = q_{21}s\tau+q_{22}c\tau \qquad \theta_2 = q_{22}s\tau-q_{21}c\tau \tag{23}$$

where

$$q_{21} = [-\lambda_1(V_2-V_3) + \lambda_2(V_4-V_1)]/[(V_2-V_3)^2 + (V_4-V_1)^2]$$

$$q_{22} = [\ \lambda_1(V_4-V_1) + \lambda_2(V_2-V_3)]/[(V_2-V_3)^2 + (V_4-V_1)^2]$$

$$\lambda_1=2db_{23} \quad \lambda_2=2db_{13} \quad q_{11}=q_{21}N_2-q_{22}N_1 \quad q_{12}=q_{21}N_1+q_{22}N_2$$

$$V_1 = N_2-U_5 \quad V_2 = N_1-U_7 \quad V_3 = N_1U_6 \quad V_4 = U_6(1+N_2)$$

$$N_1 = (U_1U_4+U_2U_3)/(U_1^2+U_3^2) \qquad N_2 = (U_3U_4-U_1U_2)/(U_1^2+U_3^2) \tag{24}$$

and

$$U_1 = k_o-1 \qquad U_2 = -(a+h_{01}) \qquad U_3=k_1 \quad U_4=h_{11}$$

$$U_5 = h_{02}-e_1 \qquad U_6 = e_2 \qquad U_7=h_{12} \tag{25}$$

For small misalignments b_{13}, b_{23} of the drive axis of B, Eqs.(22) and (23) give first order approximations to satellite motion for either rigid interconnection (h_{01}, h_{02} very large) or low coupling interconnection (h_{01}, h_{02}, h_{11}, h_{12}, and a very small) between A and B (See Eq.(13)).

Discussion of Solution

Rigid Interconnection:

For small amplitude motions, the angle ψ_r which the symmetry axis of A makes with its nominal direction (that for which $\phi_1=\phi_2=0$) is accurately given by

$$\psi_r = (\phi_1^2+\phi_2^2)^{1/2} \tag{26}$$

and taking h_{01} and h_{02} to infinity in Eq. (26) gives

$$\psi_r = \delta_r/[(1+R_0)^2 + R_1^2]^{1/2} \tag{27}$$

where

$$\delta_r = (b_{13}^2+b_{23}^2)^{1/2}\left|(B_3-B_1)/(B_3-J)\right|$$

$$R_0 = K_0/\omega^2(B_3-J) \qquad R_1 = K_1/\omega(B_3-J) \qquad J = J_1+J_2 \tag{28}$$

Since ψ_r is constant with respect to time, the symmetry axis once each period $T=2\pi/\omega$ sec traces a right circular cone with half angle ψ_r.

For satellite motion given by the solutions ϕ_1, ϕ_2, ϕ_3, the minimum magnitude $\overline{H}_r$ of CMG angular momentum necessary to provide the control torque of Eq. (1) is determined as shown in Appendix C.

$$\overline{H}_r = H_{0r}\{(R_0^2+R_1^2)/[(1+R_0)^2 + R_1^2]\}^{1/2} \tag{29}$$

where $$H_{0r} = (b_{13}^2+b_{23}^2)^{1/2}\ \omega|B_3-B_1| \tag{30}$$

Now the values of ψ_r/δ_r and H_r/H_{0r} can be plotted against R_0 for varying R_1. However, the nature of these curves depends upon whether B_3-J is greater than or less than zero. Defining

$$R_0^{\pm} = \pm R_0 \qquad R_1^{\pm} = \pm R_1 \tag{31}$$

where the positive sign is taken for $B_3-J>0$, and the negative sign is taken for $B_3-J<0$, Figs. 3 and 4 show the variation of ψ_r/δ_r and $\bar{H}_r/H_{0r}$ with respect of R_0^+ and R_1^+ and Figs. 5 and 6 show the variation of ψ_r/δ_r and $\bar{H}_r/H_{0r}$ with respect to R_0^- and R_1^-. These plots reveal the following:

1) For a satellite with $B_3-J<0$, values of K_0 in the region of $K_0=\omega^2(J-B_3)$ should be avoided.
2) For decreasing R_0^+, R_1^+ or R_0^-, R_1^- corresponding to decreasing CMG spin angular momentum, the satellite coning angle approaches

$$\psi_r = \delta_r = (b_{13}^2+b_{23}^2)^{1/2} \left|(B_3-B_1)/(B_3-J)\right| \tag{32}$$

This limiting value of $\psi_r=\delta_r$ is hereafter called the light control coning angle.

For increasing R_0^+, R_1^+ or R_0^-, R_1^-, the control moment gyros cannot have significant effect in decreasing the satellite coning angle until the magnitude of their composite spin angular momentum is of the order of the limiting value

$$\bar{H}_r = H_{0r} = \omega(b_{13}^2+b_{23}^2)^{1/2} \left|B_3-B_1\right| \tag{33}$$

Low Coupling Interconnection:

For a low coupling interconnection between bodies A and B, it can likewise be shown that the symmetry axis of A traces a right circular cone with half angle ψ_ℓ once each period $T=2\pi/\omega$ sec.

$$\psi_\ell = \delta_\ell/[(-1+k_0)^2 + k_1^2]^{1/2} + O_2(h_{01}, h_{02}, h_{11}, h_{12}, a) \quad (34)$$

where

$$\delta_\ell = (b_{13}+b_{23})^{1/2}[(h_{01}+a)^2 + h_{11}^2]^{1/2}\left|(B_3-B_1)/\bar{J}\right|, \quad \bar{J}=B_3-B_1-M\ell_B^2 \quad (35)$$

and $O_2(h_{01}, h_{02}, h_{11}, h_{12}, a)$ is a term of first order in b_{13} and b_{23} multiplied by second and higher powers of $h_{01}, h_{02}, h_{11}, h_{12}, a$ and consequently may be neglected. Also, dropping higher powers in these small parameters, the minimum magnitude $\bar{H}_\ell$ of CMG angular momentum necessary to provide the control torques of Eq. (1) is determined as shown in Appendix C.

$$\bar{H}_\ell = H_{0\ell}\{(k_0^2+k_1^2)/[(-1+k_0)^2 + k_1^2]\}^{1/2} \quad (36)$$

where

$$H_{0\ell} = \omega J_1(b_{13}^2+b_{23}^2)^{1/2}[(h_{01}+a)^2 + h_{11}^2]^{1/2}\left|(B_3-B_1)/\bar{J}\right| \quad (37)$$

The plots of ψ_ℓ/δ_ℓ and $\bar{H}_\ell/H_{0\ell}$ are similar to the plots of Fig. 5 and Fig. 6 where the ordinates are replaced by ψ_ℓ/δ_ℓ and $\bar{H}_\ell/H_{0\ell}$ and $\bar{R_0}$ and $\bar{R_1}$ are replaced by k_0 and k_1, respectively. The following observations may be made.

1) Values of K_0 in the region $K_0=\omega^2 J_1$ should be avoided

2) The satellite control system is not capable of producing significant reductions in the magnitude of the coning angle from the light control value $\psi_\ell=\delta_\ell$ unless the magnitude of the CMG composite angular momentum is at least the order of magnitude of the limiting value $\bar{H}_\ell=H_{0\ell}$.

Applications

Now the coning angle of A and control system requirements for a satellite with a low coupling interconnection are compared to those for the same satellite with a rigid interconnection. The ratio of light control coning angles is

$$\delta_\ell/\delta_r = \left|\frac{B_3-J}{\bar{J}}\right| [(h_{01}+a)^2 + h_{11}^2]^{1/2} \tag{38}$$

A corresponding ratio of minimum values of CMG composite angular momentum necessary to significantly reduce the light control coning angles is

$$H_{0\ell}/H_{0r} = [(h_{01}+a)^2 + h_{11}^2]^{1/2} J_1/\bar{J} \tag{39}$$

Consequently, for h_{01}, a, h_{11}, small quantities corresponding to small gimbal axes torque coefficients and the gimbal axes intersection located near the mass center of A or B, both the magnitude of the light control coning angle and control system requirements necessary to reduce that angle can be substantially smaller for a low coupling interconnection between the spinning and despun bodies than for a rigid interconnection.

This can be further illustrated by example. NASA has suggested for study[1(a)] a large space base that consists of a hub with a non-rotating section and a rotating section to which three spokes are attached in a Y fashion and that is spinning so as to provide an artificial gravity compartment at the extremities of one of the spokes. To within NASA's margin of error for mass estimates, the space base is assumed to be axially symmetrical with the following mass properties.

$$B_3 = 9.5\times10^8 \text{ slug-ft}^2 \qquad M_A = 1.1\times10^4 \text{ slugs}$$

$$\ell_A+\ell_B = 17 \text{ ft} \qquad M_B = 3.1\times10^4 \text{ slugs}$$

Additional data was not available. However, the spinning section B presents a flat profile and its inertia characteristics approach those of a plane figure. Hence, the following appears reasonable

$$B_1 = 4.85\times10^8 \text{ slug-ft}^2$$

and

$$A_1 = 3.2\times10^7 \text{ slug-ft}^2$$

Now due to uncertainties in construction, structural deformations, and relocations of men and equipment, misalignment of 0.5° and more of the spinning section principal axis of inertia from the spin axis appears possible.

By Eq.(32), a 0.5° misalignment results in a light control coning angle of $\psi_r=\delta_r=0.54°$. A coning angle of 0.54° is likely unacceptable. This amplitude of motion is outside the fine pointing requirements for many experiments planned for future space stations[3]. Also, for this amplitude and ω=4rpm, locations at distances greater than four feet from the space base mass center experience instantaneous accelerations greater than 10^{-4}g. Some experiments, such as those involving crystal growth, require accelerations perhaps less than 10^{-5}g.

Now if for these or other reasons the coning angle is unacceptable, then, as declared in statement 2) above, for a significant reduction the CMGs must have a momentum capacity of at least the order of H_{0r}. For ω=4rpm, Eq.(33) gives $\overline{H}_r=H_{0r}=1.69\times10^6$ ft-lb-sec, a value far beyond the capacity of existing CMGs, 2300 ft-lb-sec per CMG for the Skylab type.

Now for the same configuration designed with a low coupling interconnection, the parameter a can be made less than 4×10^{-3} by constructing the gimbal system such that the gimbal axes intersection lies one foot or less from the mass center of either body. Also, even for gimbal axes torques of the order of a thousand foot pounds, h_{01} and h_{11} are less than 0.05.

Consequently, taking $a=4\times10^{-3}$, $h_{01}=h_{11}=0.05$, $\ell_B=1$ ft and using Eq.(38), $\delta_\ell=0.068\times\delta_r=0.037°$. Also, for a significant reduction in this coning angle, the CMGs must have a momentum capacity of at least the order of $H_{0\ell}$. From Eq.(39), $H_{0\ell}=0.537\times10^{-2}\times H_{0r}=9100$ ft-lb-sec, a value which could be provided by four Skylab CMGs.

In practice values of h_{01} and h_{11} much less than 0.05 probably can be obtained. Although these potentially offer greater reductions in motion and control requirements, such small h_{01} and h_{11} are not considered here due to mathematical complications resulting from these quantities approaching the size of the measure $|\mu|$ of spin axis misalignment. Questions of stability arise because it is necessary to consider h_{01} and h_{11} as being of the order of $|\mu|$ so that terms involving these are shifted from matrix D to matrix F in Eq.(15). Then D has zero characteristic roots and the foregoing stability analysis does not apply.

Nevertheless, it seems probable that stability difficulties do not occur except for imaginary parts of the characteristic roots for matrix D of Eq.(15) in the regions of $2n\pi$, $n=\pm 1, \pm 2, \cdots$ and for system design to avoid these regions, very small values of h_{01} and h_{11} and corresponding very small motions and control requirements could be achieved. However, to verify this supposition requires more general techniques, the most practical probably being numerical Floquet analysis. Since the main purpose of this study is to demonstrate that low coupling interconnection may offer advantages over rigid interconnection, such a detailed computerized approach requiring specific choices of system parameters is beyond the scope of this work.

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Attachments
- References
- Appendices A, B, C
- Figures 1-6

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APPENDIX A

FORMULATION OF EQUATIONS OF MOTION

Neglecting gravity gradient and other environmental torques, the equations of motion of bodies A and B may be written

$$\frac{d\underline{H}^A}{dt} = {}^B\underline{T}^A + \ell_A\,\underline{a}_3 \times {}^B\underline{F}^A + {}^C\underline{T}^A \qquad M_A\frac{d}{dt}\,\underline{V}^A = {}^G\underline{F}^A + {}^B\underline{F}^A$$

$$\frac{d\underline{H}^B}{dt} = -\,{}^B\underline{T}^A + \ell_B\,\underline{c}_3 \times {}^B\underline{F}^A \qquad M_B\,\frac{d}{dt}(\underline{V}^A+\underline{V}) = {}^G\underline{F}^B - {}^B\underline{F}^A \qquad \text{(A-1)}$$

where $\underline{H}^A$, $\underline{H}^B$ are the angular momenta of A and B respectively, ${}^B\underline{T}^A$, ${}^B\underline{F}^A$ are the torque and force of B on A, ${}^G\underline{F}^A$, ${}^G\underline{F}^B$ are the forces of gravity on A and B, $\underline{V}^A$ is the inertial velocity of the mass center A* of A, $\underline{V}$ is the velocity of mass center B* of B relative to A*, and the remaining quantities are defined in the analysis section.

The two translational vector equations of motion can be solved for ${}^B\underline{F}^A$ after eliminating $\underline{V}^A$.

$${}^B\underline{F}^A = -\,\frac{M_A M_B}{(M_A+M_B)}\,\frac{d\underline{V}}{dt} + \frac{1}{(M_A+M_B)}\,(M_A\,{}^G\underline{F}^B - M_B\,{}^G\underline{F}^A) \qquad \text{(A-2)}$$

The forces of gravitational attraction are given by

$${}^G\underline{F}^A = \frac{M_A\ \nu\ \underline{\rho}_A}{R_A^2} \qquad {}^G\underline{F}^B = \frac{M_B\nu\underline{\rho}_B}{R_B^2} \qquad \text{(A-3)}$$

where ν is a gravitational constant, R_A, R_B are the magnitudes and unit vectors $\underline{\rho}_A$, $\underline{\rho}_B$ are in the directions of the vectors from A*, B*, respectively, to the mass center of the attracting body. For dimensions of the satellite very small in comparison to R_A, then

$$R_B \approx R_A \qquad \underline{\rho}_B \approx \underline{\rho}_A$$

and

$$M_A {}^G\underline{F}^B - M_B {}^G\underline{F}^A = 0 \tag{A-4}$$

to within terms of order of ℓ_A/R_A, ℓ_B/R_A so that

$${}^B\underline{F}^A = - \frac{M_A M_B}{(M_A+M_B)} \frac{d}{dt} \underline{V} \tag{A-5}$$

and

$$\frac{d\underline{H}^A}{dt} = {}^B\underline{T}^A - \ell_A \frac{M_A M_B}{(M_A+M_B)} \underline{a}_3 \times \frac{d\underline{V}}{dt} + {}^C\underline{T}^A \tag{A-6}$$

$$\frac{d\underline{H}^B}{dt} = - {}^B\underline{T}^A - \ell_B \frac{M_A M_B}{(M_A+M_B)} \underline{c}_3 \times \frac{d\underline{V}}{dt} \tag{A-7}$$

The angular velocities $\underline{\omega}^A$ of A and $\underline{\omega}^B$ of B can be shown to have the form

$$\underline{\omega}^A = (\dot{\phi}_1 c\phi_2 c\phi_3 + \dot{\phi}_2 s\phi_3)\underline{a}_1 + (-\dot{\phi}_1 c\phi_2 s\phi_3 + \dot{\phi}_2 c\phi_3)\underline{a}_2 + (\dot{\phi}_1 s\phi_2 + \dot{\phi}_3)\underline{a}_3$$

$$= \alpha_1\underline{a}_1 + \alpha_2\underline{a}_2 + \alpha_3\underline{a}_3 \tag{A-8}$$

$$\underline{\omega}^B = \underline{\omega}^A + \dot{\theta}_1\underline{a}_1 + \dot{\theta}_2\underline{c}_2 + \omega\underline{b}_3' \tag{A-9}$$

$$= \beta_1\underline{b}_1 + \beta_2\underline{b}_2 + \beta_3\underline{b}_3 \tag{A-10}$$

where β_1, β_2, β_3 are determined by resolving the right side of Eq.(A-9) into components along principal axes of B. Now $\underline{H}^A$ and $\underline{H}^B$ may be written

$$\underline{H}^A = A_1\alpha_1\underline{a}_1 + A_1\alpha_2\underline{a}_2 + A_3\alpha_3\underline{a}_3$$

$$\underline{H}^B = B_1\beta_1\underline{b}_1 + B_1\beta_2\underline{b}_2 + B_3\beta_3\underline{b}_3 \qquad \text{(A-11)}$$

Also, $\underline{V}$ may be written

$$\underline{V} = \ell_A\underline{\omega}^A \times \underline{a}_3 + \ell_B\underline{\omega}^B \times \underline{c}_3 \qquad \text{(A-12)}$$

The reaction torque ${}^B\underline{T}^A$ of B and A is given by

$${}^B\underline{T}^A = {}^G\underline{T}^A + L\ \underline{c}_3 \qquad \text{(A-13)}$$

where ${}^G\underline{T}^A$ is given by Eq.(3) and L is an unknown component in the direction of the spin axis of B. Substitution of Eqs.(A-8) - (A-13) into Eqs.(A-6) and (A-7) yields six equations of motion in six unknowns ϕ_1, ϕ_2, ϕ_3, θ_1, θ_2, L.

APPENDIX B

PERIODIC SOLUTIONS TO A SET OF DIFFERENTIAL EQUATIONS

Coddington and Levinson[5] have set forth sufficient conditions for existance and asymptotic stability of periodic solutions to a set of differential equations of the form

$$x' = f(\tau, x, \mu) \qquad \text{(B-1)}$$

where x is a column vector with n components, f is periodic in τ of period T, and μ may be a real constant vector. In addition, a method of successive approximations is established for determining these periodic solutions.

Following is a brief outline of the discussions of Ref.(5) with a slight modification in the statement of the method of successive approximations allowing μ to be a column vector with two components rather than a scaler.

For each τ, let f be analytic (i.e., each f_i, $i=1,\cdots,n$ is representable by a convergent power series) in (x,μ) for $|\mu|$ small and consider the case where Eq.(B-1) with $\mu=0$ has a solution p of period T. Defining the first variation of Eq.(B-1)

$$y' = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j}(\tau, p(\tau), 0) y_j = f_x(\tau, p(\tau), 0) y \qquad \text{(B-2)}$$

then,

Theorem 1 If Eq.(B-2) has no solution of period T other than the trivial solution y=0, then for small $|\mu|$, Eq.(B-1) has a unique solution $q=q(\tau,\mu)$ periodic in τ of period T with $q(\tau,0)= p(\tau)$.

Also, it may be shown that q is analytic in μ for small $|\mu|$ and any τ. Then, $q(\mu,\tau)$ has a convergent power series representation which for μ a column vector with dimension two $\mu = \begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix}$, has the form

$$q(\tau,\mu) = p^{(0)}(\tau) + \mu_1 p^{(1,1)}(\tau) + \mu_2 p^{(1,2)}(\tau) + \cdots \qquad \text{(B-3)}$$

Also, by assumption $f(\tau,q(\tau,\mu),\mu)$ has a power series representation and substituting this together with Eq.(B-3) into Eq.(B-1) and equating powers of μ_1, μ_2, there results

$$\frac{dp^{(0)}}{d\tau}(\tau) = f(\tau,p^{0}(\tau),0)$$

$$\frac{dp^{(1,1)}}{d\tau}(\tau) = f_x(\tau,p^{(0)}(\tau),0)\,p^{(1,1)}(\tau) + \frac{\partial f}{\partial \mu_1}(\tau,p^{(0)}(\tau),0)$$

$$\frac{dp^{(1,2)}}{d\tau}(\tau) = f_x(\tau,p^{(0)}(\tau),0)\,p^{(1,2)}(\tau) + \frac{\partial f}{\partial \mu_2}(\tau,p^{(0)}(\tau),0) \qquad \text{(B-4)}$$

$$\vdots$$

where $p^{(0)}(\tau) = p(\tau)$. Since q is known to exist as an analytic function of μ, the system (B-4) provides that solution. Sufficient conditions for stability of q follow:

Theorem 2 If the real parts of the characteristic exponents of Eq.(B-2) are all negative, then the solution $q=q(\tau,\mu)$ of Eq.(B-1) is asymptotically stable provided $|\mu|$ and $|x|_{\tau=0}$ are small.

APPENDIX C

ANGULAR MOMENTUM REQUIREMENTS FOR CMGs

The equations of motion for the CMG configuration may be written in terms of the configuration composite spin angular momentum $\underline{H}^C$, output torque ${}^C\underline{T}^A$, and angular velocity $\underline{\omega}^A$ of body A.

$$\dot{\underline{H}}^C + \underline{\omega}^A \times \underline{H}^C = - {}^C\underline{T}^A \tag{C-1}$$

Or, written in terms of components associated with the directions of the three CMG output axes

$$\dot{H}_1 + \alpha_2 H_3 - \alpha_3 H_2 = K_1 \dot{\phi}_1 + K_0 \phi_1 \tag{C-2}$$

$$\dot{H}_2 + \alpha_3 H_1 - \alpha_1 H_3 = K_1 \dot{\phi}_2 + K_0 \phi_2 \tag{C-3}$$

$$\dot{H}_3 + \alpha_1 H_2 - \alpha_2 H_1 = K_{13} \dot{\phi}_3 + K_{03} \phi_3 \tag{C-4}$$

Now the value of the α_i component of $\underline{\omega}^A$ is equal to $\dot{\phi}_i$, i=1,2,3, to the first power in b_{13},b_{23} and substitution of $\dot{\phi}_i$ from Eq.(22) for α_i in Eqs.(C-2)-(C-4) yields

$$\dot{H}_1 + \dot{\phi}_2 H_3 = K_1 \dot{\phi}_1 + K_0 \phi_1 \tag{C-5}$$

$$\dot{H}_2 - \dot{\phi}_1 H_3 = K_1 \dot{\phi}_2 + K_0 \phi_2 \tag{C-6}$$

$$\dot{H}_3 + \dot{\phi}_1 H_2 - \dot{\phi}_2 H_1 = 0 \tag{C-7}$$

Multiplying Eq.(C-5) by $\dot{\phi}_1$, Eq.(C-6) by $\dot{\phi}_2$, and adding the result, it can be shown using Eqs.(15) and (16) that

$$\dot{H}_1\dot{\phi}_1 + \dot{H}_2\dot{\phi}_2 = \omega^2(b_{13}^2+b_{23}^2)P \tag{C-8}$$

where P is a term involving R_0 and R_1 or k_0 and k_1, which implies that H_1 and H_2 are proportional to b_{13},b_{23} and, by Eq.(C-7), H_3 is proportional to terms of second order in b_{13},b_{23}. Then, to the first power in b_{13},b_{23},

$$\dot{H}_i = K_1\dot{\phi}_i + K_0\phi_i \qquad i=1,2 \qquad \dot{H}_3 = 0 \tag{C-9}$$

Now Eq.(C-9) can be integrated using Eq.(22) and considering the mean value of H_1, H_2, H_3 to be zero in order to minimize $|\underline{H}^C|$, the minimum magnitude $\overline{H}$ of angular momentum is then given by

$$\overline{H} = (H_1^2 + H_2^2 + H_3^2)^{1/2} \tag{C-10}$$

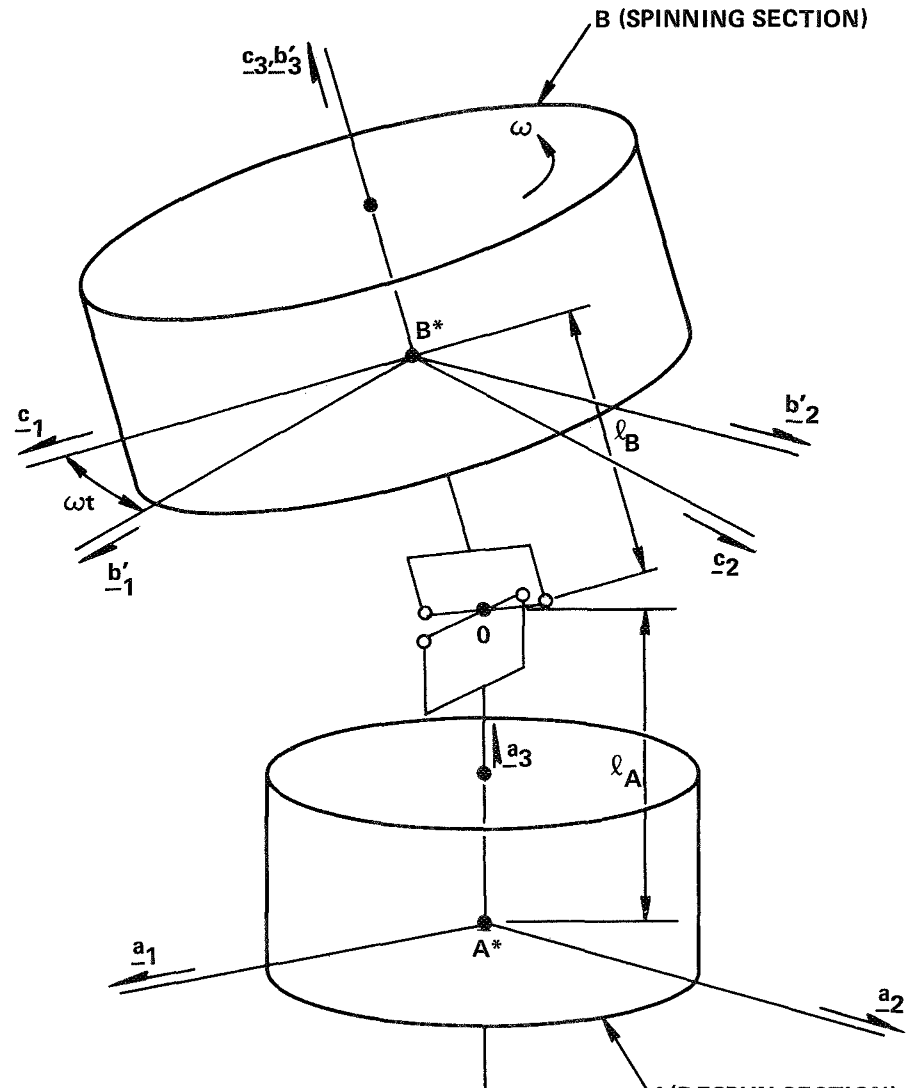

FIGURE 1 - DUAL SPIN SATELLITE – FLEXIBLE INTERCONNECTION

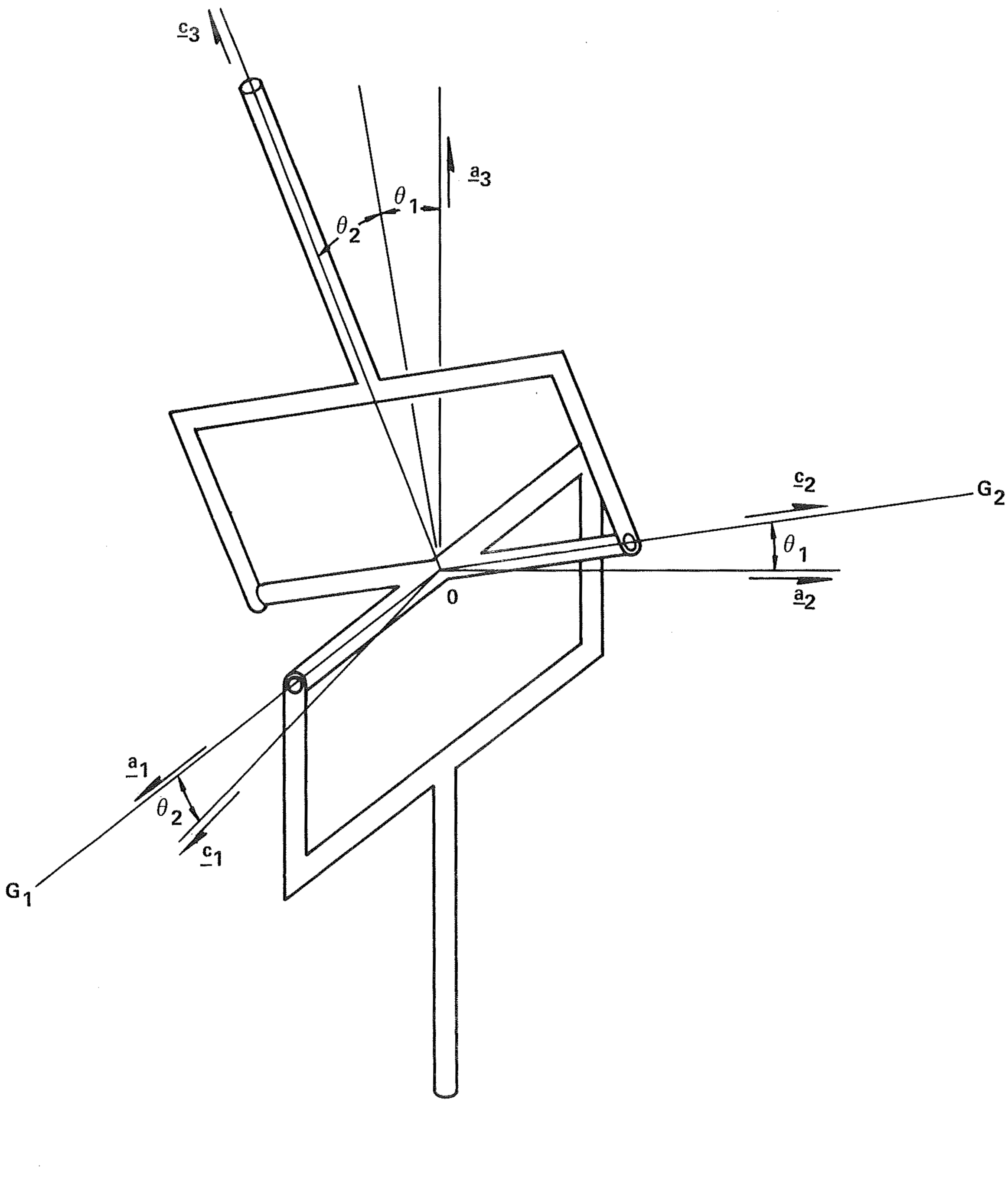

FIGURE 2 - MODEL OF FLEXIBLE INTERCONNECTION

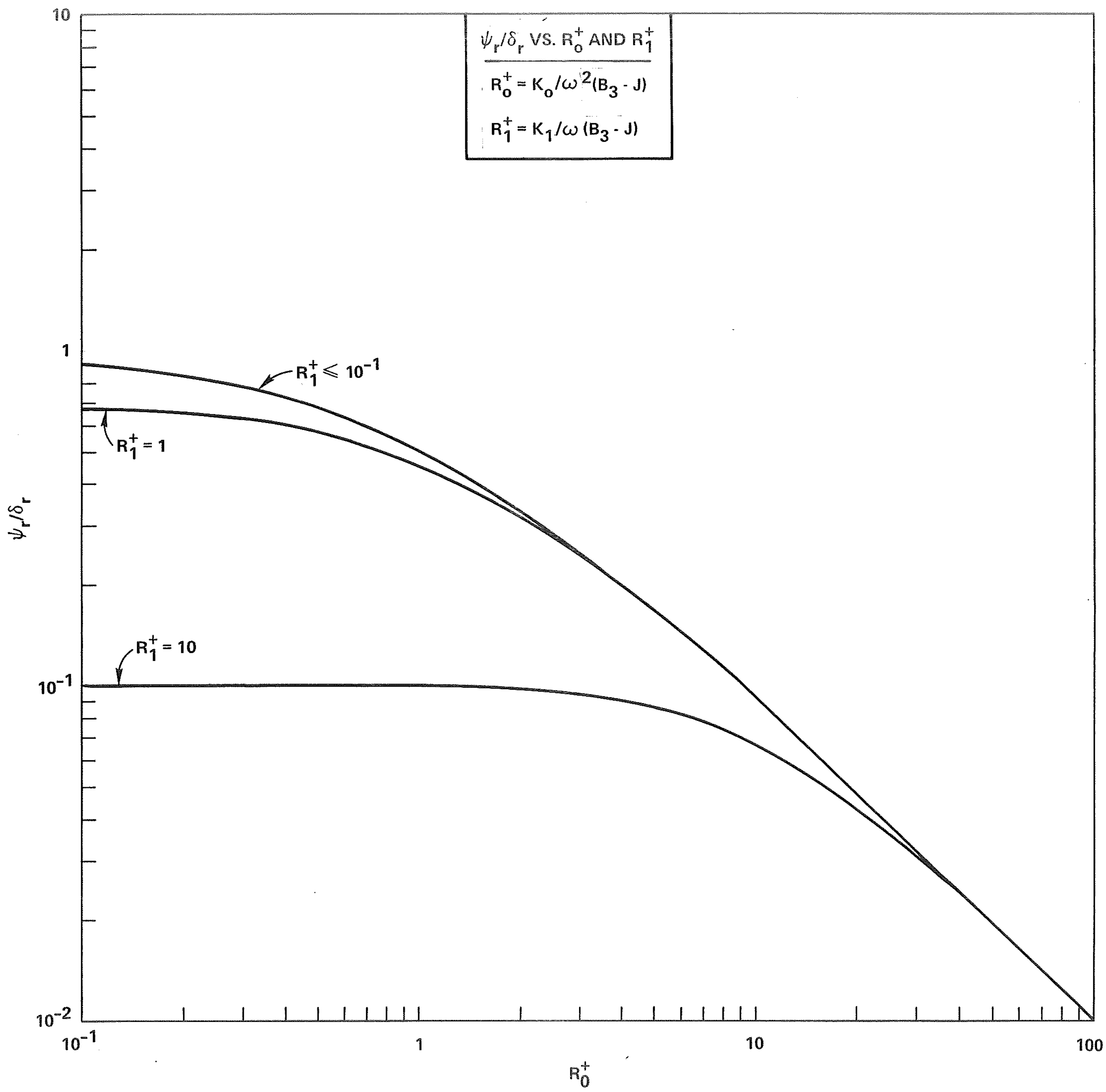

FIGURE 3. SPACE STATION CONING ANGLE FOR $B_3 > J$

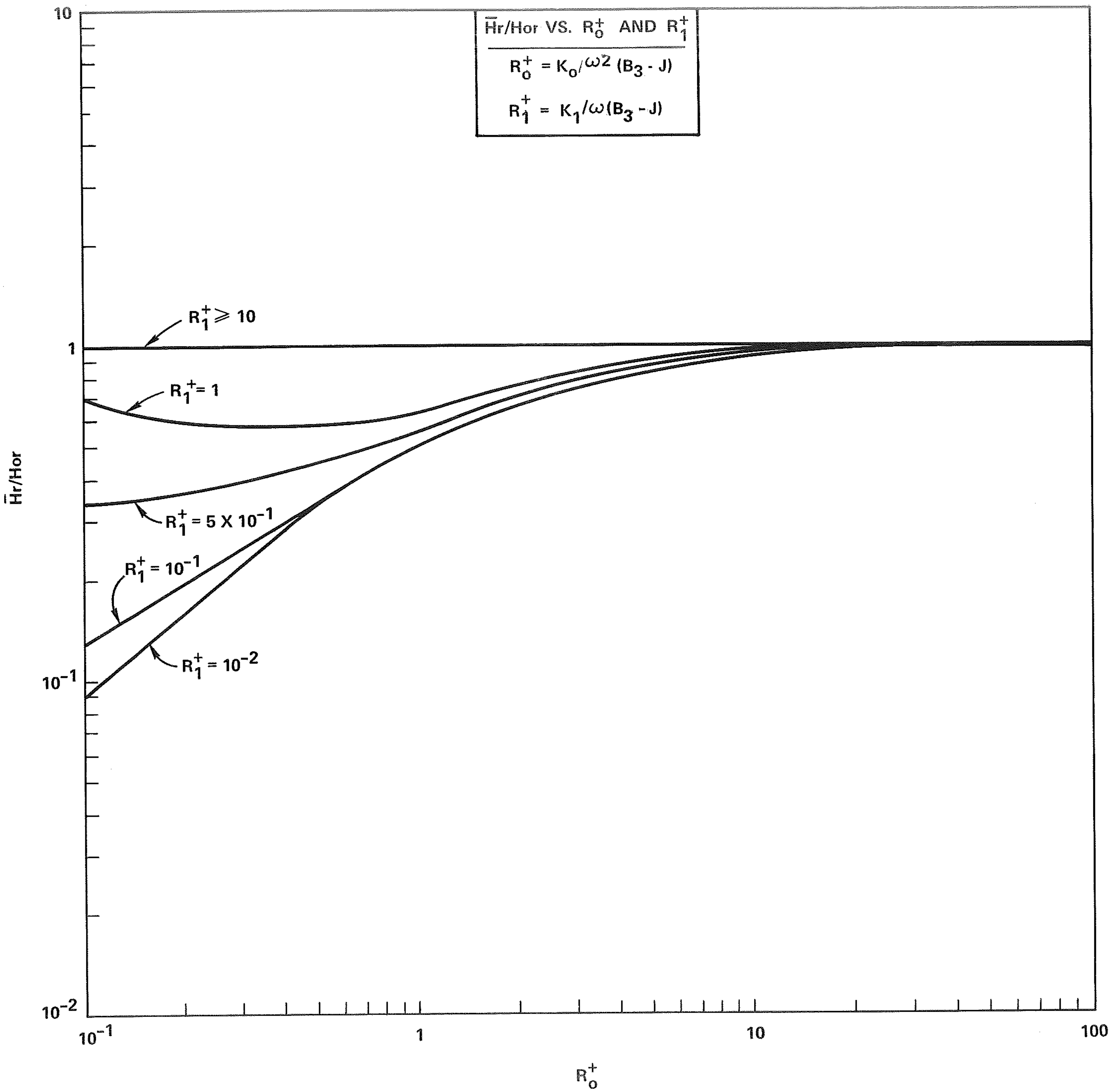

FIGURE 4. MINIMUM CMG ANGULAR MOMENTUM FOR $B_3 > J$

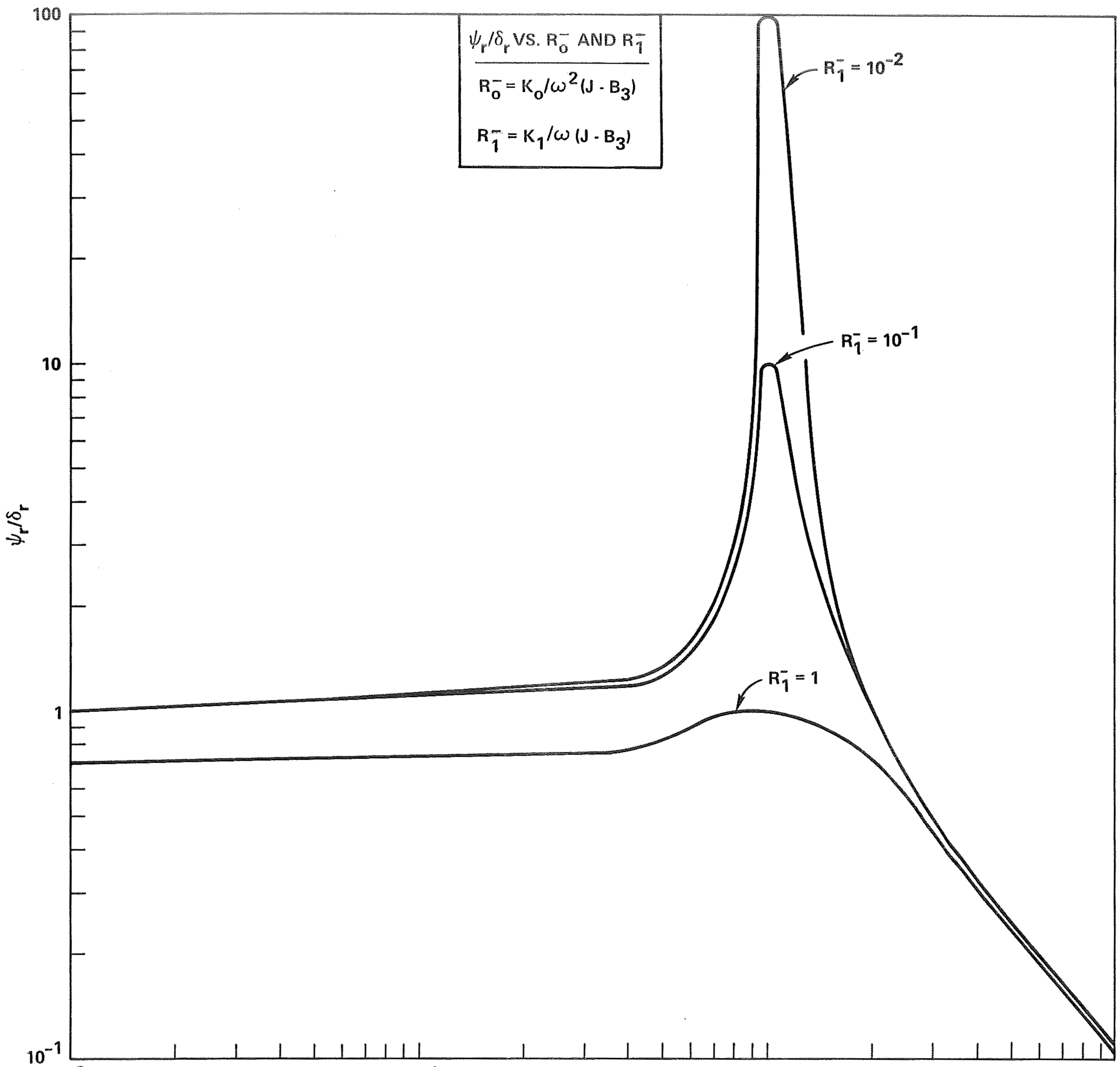

FIGURE 5. SPACE STATION CONING ANGLE FOR $B_3 < J$

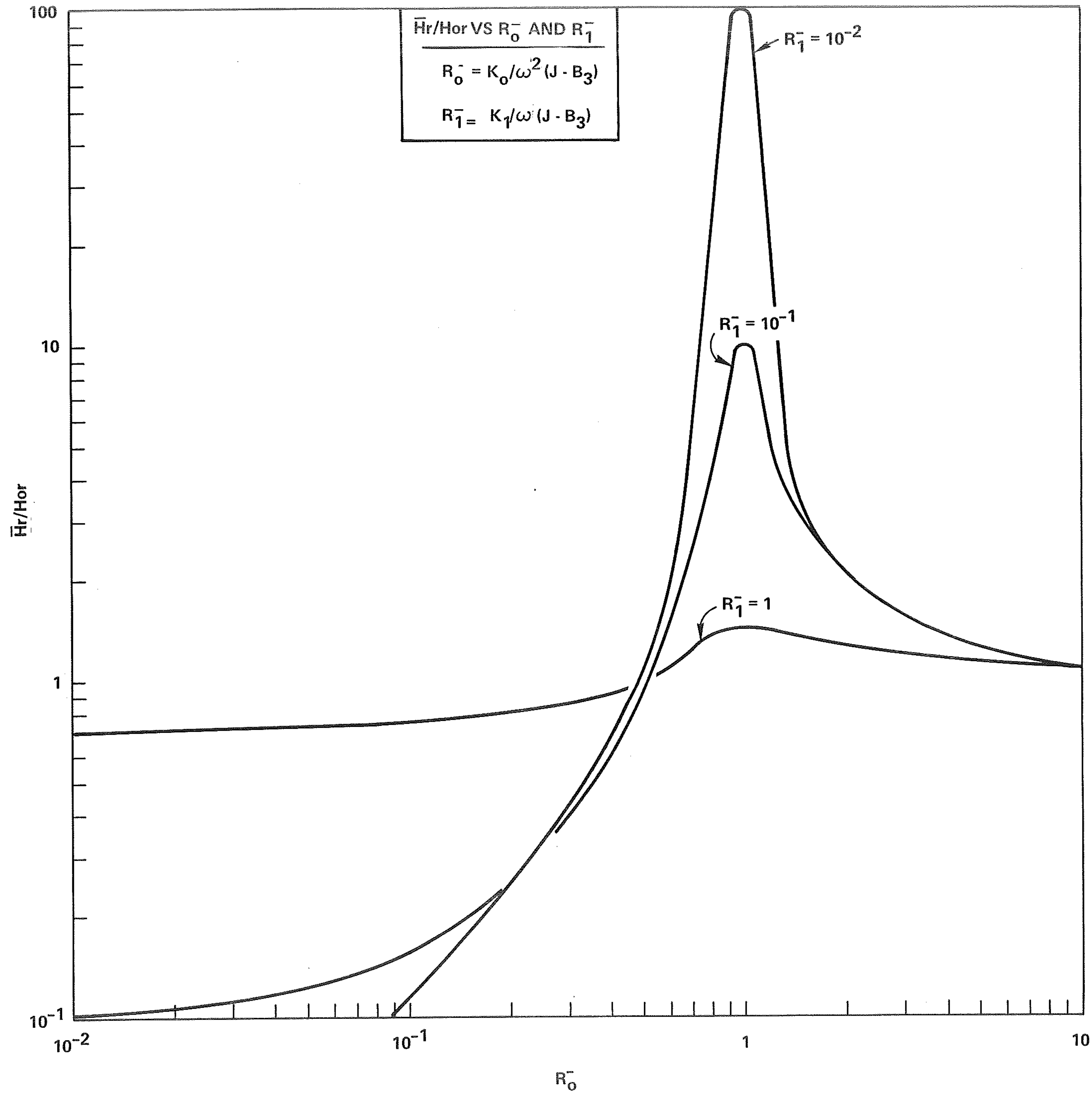

FIGURE 6. MINIMUM CMG ANGULAR MOMENTUM FOR $B_3 < J$